ÉRETTSÉGI VIZSGA ● 2010. május 4.

MATEMATIKA
ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI
ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI
ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS
MINISZTÉRIUM

Important Information

Formal requirements:

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

Substantial requirements:

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 4 questions to be marked out of the 5 in part II of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student’s request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

I.

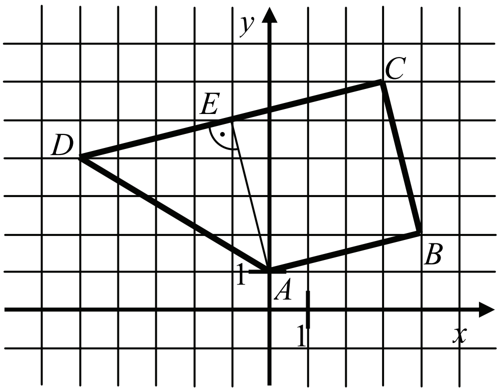
1. a) **first solution**		
The l.h.s. of the first equation (on the domain) is $\log_2\left(xy^3\right)=\log_2 x+3\log_2 y$.	1 point	
The l.h.s. of the second equation is $\log_2\left(x^2y\right)=2\log_2 x+\log_2 y$.	1 point	
The system is hence $\log_2 x+3\log_2 y=1$ $2\log_2 x+\log_2 y=-3$. Subtracting the second equation from the double of the first one yields $5\log_2 y=5\Leftrightarrow \log_2 y=1$,	1 point	
therefore $y=2$.	1 point	
Substituting $\log_2 x=-2$,	1 point	
and thus $x=2^{-2}=\frac{1}{4}$.	1 point	
The positive values hence obtained satisfy the system. (checking).	1 point	
Total:	**7 points**	

1. a) **second solution**		
By the definition of common logarithms $\log_2\left(xy^3\right)=1\Leftrightarrow xy^3=2$ (on the domain)	1 point	
$\log_2\left(x^2y\right)=-3\Leftrightarrow x^2y=\frac{1}{8}$.	1 point	
From the second equation $y=\frac{1}{8x^2}$,	1 point	*These 2 points are also due if the candidate arrives to $x^5=\frac{1}{2^{10}}$ by dividing the cube of the second equation by the first one or – dividing the square of the first equation by the second one – to $y^5=2^5$.*
and substituting into the first equation one gets $\frac{1}{512x^5}=2$,	1 point	
and thus $x=2^{-2}=\frac{1}{4}$.	1 point	
Substituting $y=2$.	1 point	
The positive values hence obtained satisfy the system (checking).	1 point	
Total:	**7 points**	

1. b)		
Let n be the positive integer value of the given expression. $n = \log_{3^k} 729 = \log_{3^k} 3^6$,	1 point	
that is $\left(3^k\right)^n = 3^6$.	1 point	
Since k and n are positive integers k is a positive divisor of 6.	1 point	
The possible values of k are hence 1, 2, 3, 6.	2 points	
Total:	**5 points**	

1. *If the candidate finds just two or three correct values of k then 1 point should be given from the last 2. No point may be given if there is just one value of k found.*
2. *If the candidate calculates the value of* $\log_{3^k} 729$ *for the first six positive integer values of k but fails to show that it is not a whole number if k is greater than 6 (in fact it is a positive number less than 1) then at most 3 poins may be given.*

2. a) first solution



Since $\overrightarrow{AB}(4;1)$,	1 point	*If the candidate identifies the trapezium from the diagram without any calculations then 1 point may be given.*
$\overrightarrow{DC}(8;2)$,	1 point	
therefore $\overrightarrow{DC} = 2\overrightarrow{AB}$.	1 point	
Hence *AB* and *DC* are parallel, the quadrilateral *ABCD* is a trapezium, indeed.	1 point	

2. a) second solution

Since $\overrightarrow{AB}(4;1)$,	1 point	*If the candidate identifies the trapezium from the diagram without any calculations then 1 point may be given.*
$\overrightarrow{DC}(8;2)$,	1 point	
and thus $\overrightarrow{DC} = 2\overrightarrow{AB}$.	1 point	
Therefore, *AB* and *DC* are parallel, the quadrilateral *ABCD* is a trapezium indeed.	1 point	

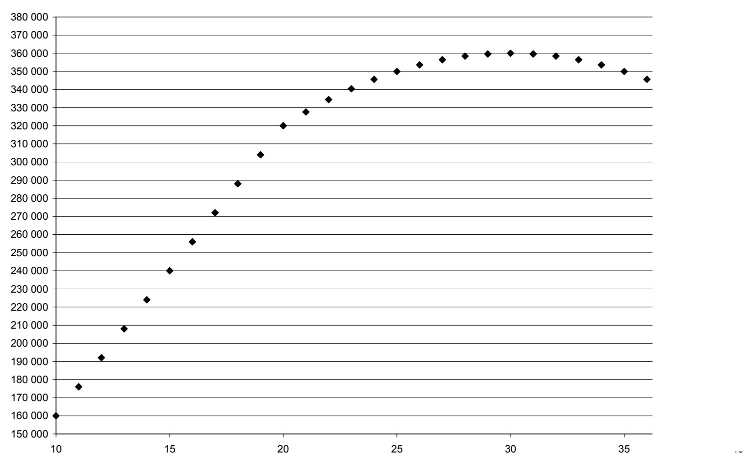
2. b)

A complete graph on *n* vertices has $\frac{n(n-1)}{2}$ edges.	1 point	*This point is due even if this idea appears implicitely in the soltution.*
$\frac{n(n-1)}{2} = 253$;	1 point	
$n^2 - n - 506 = 0$.	1 point	
$n_1 = 23$;	1 point	
$n_2 < 0$, it is not a solution to the problem.	1 point	
The graph has 23 vertices, and thus there are 19 new vertices to be inserted.	1 point	
A connected graph on *n* vertices has at least n-1 edges,	1 point	
therefore, at most 231 edges may be deleted.	1 point	
Total:	**12 points**	

3. a)		
If $10 \le x \le 20$ (x integer) then $P(x) = 16000x$.	1 point	
If $20 < x \le 36$ (x integer) then the discount is $400 \cdot (x - 20)$ per ticket,	1 point	
and thus $P(x) = 16000x - 400 \cdot (x - 20) \cdot x = 400x(60 - x)$.	2 points	
Summarizing $P(x) = \begin{cases} 16000x, & \text{if } 10 \le x \le 20 \ (x \text{ integer}) \\ 400x(60 - x), & \text{if } 20 < x \le 36 \ (x \text{ integer}) \end{cases}$	1 point	*This point is due even if the candidate does not write down the single formula for the function but still obtains the correct fromulas separately for the two intervals.*
The domain is $x \in \mathbf{N}$; $10 \le x \le 36$.	1 point	
Total:	**6 points**	
If the domains of the respective formulas are not mentioned throughout the solution then at most 3 points may be given		

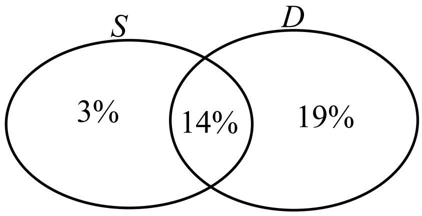
3. b)		
The income function $P(x)$ is linear on the interval $10 \le x \le 20$ and it is strictly increasing, therefore, it assumes its maximum at $x = 20$ on this inteval. $P(20) = 320000$.	1 point	
If $20 < x \le 36$ then the fromula for the income function is $400x(60 - x) = -400x^2 + 24000x =$ $= -400(x - 30)^2 + 360000$	2 points	*These 2 points cannot be split further.*
$-400(x-30)^2 + 360000$ asumes its maximum if $x = 30$.	1 point	
Since $20 < 30 \le 36$ and 30 is a whole number, the profit function has a local maximum at 30.	1 point	
The value of the local maximum is 360 000 Ft.	1 point	*If the candidate finds the extremum point and the extremal value by differentiation or as the midpoint of the roots of the quadratic but does not mention that this method works only in case of the smooth extension of the discreet function then at most 3 points of the 5 may be given.*
The income will be maximal if there are 30 passengers since $P(20) < P(30)$); the maximal income is then 360 000 Ft.	1 point	
Total:	**7 points**	

If the candidate is tabulating the function and indicates its values for every integer value of the domain and the maximal income with the corresponding number of passengers is found from the table then full score should be given for questions a) and b). Accordingly, full score is due if the candidate sketches the correct graph and the relevant information is correctly assessed from here. If, however, the solution is incomplete then the score should be reduced according to the marking scheme.



4. a)		
Let S and D be the events of shopping on the net or downloading software, respectively. Since $P(S) = 0.17$,	1 point	*These points are due if these ideas correctly appear in the solution.*
the probability of the complementary event is:	1 point	
$P(\overline{S}) = 1 - P(V) = 0.83$.	1 point	
Total:	**3 points**	
The correct result based on Venn-diagrams is also worth 3 points.		

4. b) first solution		
The question is the probability of the event $S+D$.	1 point	*These points are due if these ideas correctly appear in the solution.*
Since $P(S+D) = P(S) + P(D) - P(SD)$,	1 point	
and $P(SD)$, = 0.14.	1 point	
Thus $P(S+D) = 0.17 + 0.33 - 0.14 = 0.36$ (36 %).	1 point	
Total:	**4 points**	

4. b) second solution		
The sets of the shopping and the downloading users can be represented on a Venn-diagram: 	1 point	
The probability of belonging to the set $S \cup D$, that is	1 point	*These points are due if these ideas correctly appear in the solution.*
$P(S$ or $D) = P(S) + P(D) - P(S$ and $D)$ is given as	1 point	
$P(S + D) = 0.36$ (36 %).	1 point	
Total:	**4 points**	

4. c)		
This event is the complement of the previous one, therefore	2 points	
$P(\overline{S}\overline{D}) = 1 - 0.36 = 0.64\ (64\%)$.	1 point	
Total:	**3 points**	

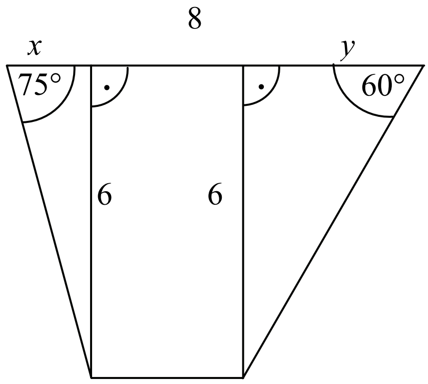
4. d)		
The three users do not do shopping independently,with a probability of 0.83 each,	2 points	
therefore P(none of them does shopping) = $0.83^3 \approx$	1 point	
≈ 0.57. (57%.)	1 point	
Total:	**4 points**	

II.

5.		
Let the number of boys be *b*. The sum of their indivual results is then 4,01*b*.	1 point	
If there were *g* girls then the sum of their indivual results is 4,21*g*. $(b; g \in \mathbf{N}^{+}.)$	1 point	
The size of the total student body is *b* + *g*. the sum of their indivual results is 4,12 (*b* + *g*).	1 point	
4,01*b* + 4,21*g* = 4,12 (*b* + *g*).	2 points	
Rearranging $g = \frac{11}{9}b$.	2 points	
The total is $b + g = b + \frac{11}{9}b = \frac{20}{9}b$.	1 point*	
Being a whole number *b* + *g* must be divisible by 9.	2 points*	
By condition $400 < \frac{20}{9}b < 430$.	1 point*	*The total, of course, can be expressed in terms of g:* *(* $b = \frac{9}{11}g$, $b + g = \frac{20}{11}g$, $220 < g < 236{,}5$ *)* *This point is due if the candidate notes at the beginning that in case of x students* 400 < *x* < 430.
Hence $180 < b < 193{,}5$.	1 point*	
Since *b* is divisible by 9, therefore, *b* = 189.	1 point*	
g = 231.	1 point*	
The total number of students is hence 189 + 231 = 420.	1 point*	
Checking according to the text of the problem.	1 point	
Total:	**16 points**	

*The altogeteher 8 points marked by * can be also given for the following argument: if the proportion between the number of boys and girls is 9:11, then the total number of students is divisible by 20.* (4 points).
Since there 420 is the only multiple of 20 between 400 and 430 (2 points)*,*
there are 420 students in the school. (2 points).

6. a)



The diagram shows the required cross section of the ditch.	4 points	*A good diagram without data is worth 2 points.*
Total:	**4 points**	

6. b) first solution

The ditch is a straight prism whose base is congruent to the above trapezium.	2 points	
The height of this prism is 8 meter.	1 point	
The area of the trapezium (base) is $A=\frac{(8-x-y)+8}{2}\cdot 6$	2 points	
$\tan 60° = \frac{6}{x}$.	1 point	
$x=\frac{6}{\tan 60°}=\frac{6}{\sqrt{3}}\ (\approx 3{,}46)$.	1 point	
$\tan 75° = \frac{6}{y}$.	1 point	
$y=\frac{6}{\tan 75°}\left(=\frac{6}{2+\sqrt{3}}\approx 1.61\right)$.	1 point	
$A\approx\frac{8-3.46-1.61+8}{2}\cdot 6\ (=32.79)$.	1 point	
$V\approx 32.79\cdot 8\approx 262.3$.	1 point	
The amount of soil to be dug out is 262 m^3 .	1 point	
Total:	**12 points**	*The result is $262\,m^3$ even if the candidate is using more accurate partial results.*

6. b) **second solution**		
The solid can be completed into a brick of dimensions 8×8×6 meter whose volume hence must be decreased by the volumes of the respective triangular pyramids adjacent to the slant faces.	1 point	
The volume of the brick is $8 \cdot 8 \cdot 6 = 384$ (m^3).	1 point	
$\tan 60° = \frac{6}{x}$.	1 point	
$x = \frac{6}{\tan 60°} \left(= \frac{6}{\sqrt{3}} \approx 3.46 \right)$.	1 point	
$\tan 75° = \frac{6}{y}$.	1 point	
$y = \frac{6}{\tan 75°} \left(= \frac{6}{2+\sqrt{3}} \approx 1.61 \right)$.	1 point	
The base of one of the adjoined triangular pyramids is a right triangle. One of its legs is 6, and its hypotenuse is $x \approx 3.46$. The height if the pyramid is 8.	1 point	
Its volume is hence $V_1 \approx \frac{6 \cdot 3{,}46}{2} \cdot 8 = 83.04$ (m^3).	1 point	
The base of the other adjoined triangular pyramid is also a right triangle. One of its legs is 6, and the other leg is $y \approx 1.61$. The height of this pyramid is also 8.	1 point	
Its volume is hence: $V_2 \approx \frac{6 \cdot 1.61}{2} \cdot 8 = 38.64$ (m^3).	1 point	
$V \approx 384 - 83.04 - 38.64 \approx 262.3$,	1 point	
The amount of soil to be dug out is 262 m^3 .	1 point	
Total:	**12 points**	
As in the second solution, one can also proceed by dividing the given solid into a brick and two triangular pyramids. Its assesment should follow the marking scheme of the second solution.		

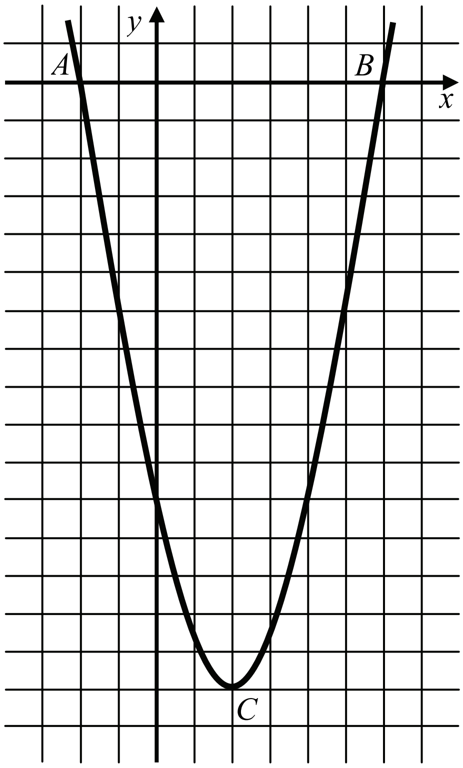
7. a)		
There are $\binom{30}{5}$ ways to choose 5 out of 30 students.	1 point	
In the problem there are 2 students selected out of 12 and, independently, there are 3 more selected out of the remaining 18. There are $\binom{12}{2}\cdot\binom{18}{3}$ ways to do that.	1 point	
The probability that there are exactly two students having revision class is hence $P(A_2)=\dfrac{\binom{12}{2}\cdot\binom{18}{3}}{\binom{30}{5}}=$	1 point	
$\left(=\dfrac{66\cdot 816}{142506}\right)=\dfrac{53856}{142506}\approx 0.378$.	1 point	*Any decimal form, like 0.3779 or 0.38. of the result should be accepted,*
Total:	**4 points**	

7. b) first solution		
Denote the event that there are some among the chosen students who have revision class by B, and the event that there are exactly i ones among the chosen students having revision class by A_i.		
The conditional probability $P(A_2\|B)$ has to be computed.	1 point	
$\mathrm{P}(A_2\|B) = \dfrac{\mathrm{P}(A_2B)}{\mathrm{P}(B)} = \dfrac{\mathrm{P}(A_2)}{\mathrm{P}(B)}$.	1 point	
P(B) can be calculated in terms of the probability of the complementary event as $\mathrm{P}(B) = 1 - \mathrm{P}(\overline{B})$.	1 point	
Since $\overline{B} = A_0$, therefore	1 point	
$\mathrm{P}(B) = 1 - \dfrac{\binom{12}{0}\cdot\binom{18}{5}}{\binom{30}{5}} \approx$	1 point	
$\approx 1 - 0.0601 \approx 0.940$.	1 point	*Any other rounded result should be accepted.*
The probability is hence equal to $\dfrac{0.378}{0.940} \approx 0.402$.	1 point	*The fraction form – or its variations like* $\dfrac{378}{940}$ *may also be accepted.*
Total:	**7 points**	*Maximal score is due even if the candidate does not write down the general formula for conditional probability but still uses it correctly.*

7. b) second solution		
There are $\binom{30}{5} - \binom{18}{5}$ cases when there is some student among the chosen ones that has revision class.	2 points	
This is equal to 133 938.	1 point	
Each of these outcomes are equally probable.	1 point	
There are $\binom{12}{2}\cdot\binom{18}{3} = 53\,856$ out of these 133 938 outcomes when there are exactly two selected students have revision classes.	2 points	
The probablity is hence equal to $\dfrac{53856}{133938}\left(\approx 0.402\right)$.	1 point	
Total:	**7 points**	

7. c)		
If the winner team has scored v goals , then the loser has scored $v + 3$ goals. The total number of goals is hence $2v + 3$.	1 point	
According to the spectators $4 < v$, and $10 < 2v + 3 < 28$. Hence $4 < v \leq 12$. Taking into account E's statement the possible values of v are 5, 7 or 11.	1 point	
D says that $2v + 3$ is a prime number; if $v = 5$ then $2v + 3 = 13$ is a prime, indeed.	1 point	
If $v = 7$ then $2v + 3 = 17$ also a prime.	1 point	
(If $v = 11$ then $2v + 3 = 25$ not a prime number. Therefore, the result of the match cannot be deduced from the given information; both 8:5 and 10:7 are consistent.	1 point	
Total:	**5 points**	
These 5 points are due also if the candidate prepares a table, enters the data of the question and properly arrives to the correct answer, i.e. that there are two possible results. However, if only the two possible results are written down with no explanation whatsoever, then it is just the last 1 point may be given.		

8.		
Let $a_1 = a$. Then by (1) $b_1 = 2a$, $c_1 = 4a$.	1 point	*Since the third terms of the first and the second progression are not needed for further computations, these 5 points are due even if these terms are not written down here.*
Denote the common ratio of $\{a_n\}$ by q. Then, by (2) the common ratio of $\{b_n\}$ is $q + 1$ and that of $\{c_n\}$ is $q + 2$.	1 point	
Hence the first three terms of the three progressions a aq aq^2	1 point	
$2a$ $2a(q + 1)$ $2a(q + 1)^2$	1 point	
$4a$ $4a(q + 2)$ $4a(q + 2)^2$, respectively	1 point	
The following also hold (3) $aq + 2a(q + 1) + 4a(q + 2) = 24$ and	1 point	*5 points are due for the corect quadratic equation.*
(4) $4a + 4a(q + 2) + 4a(q + 2)^2 = 84$.	1 point	
With a bit of algebra one gets the following simultaneous system: $7aq + 10a = 24$.	1 point	
$4a(q^2 + 5q + 7) = 84$.	1 point	
Isolating a from the equations and make the expressions equal yields $8q^2 - 9q - 14 = 0$.	1 point	
Its solutions are $q_1 = 2$ and $q_2 = -\frac{7}{8}$.	1 point	
In the first case $a = 1$.	1 point	
In the second case $a = \frac{192}{31}$, not an integer and thus it is not a solution.	1 point	*If the candidate accepts this solution and writes down thecorresponding three sequences then this point and the 1 for the checking (2 altogether) cannot be given*
The first three terms of the sequences are 1 2 4 2 6 18 4 16 64 respectively.	2 points	*If the candidate finds these solutions by intelligent guessing and does not show that there is no other solution then at most 8 points may be given.*
These values satisfy the conditions of the question.	1 point	
Total:	**16 points**	

9. a) **first solution**		
		
From the equation of the parabola: $y = x^2 - 4x - 12 = (x-2)^2 - 16 = (x+2)(x-6)$.	1 point	
Triangle *ABC* is isosceles, its vertices can be deduced from the equation of the parabola: *A*(–2; 0), *B*(6; 0), *C*(2; –16).	1 point	
Accordingly, $AB = 8$ and the altitude to the base *AB* is $h = 16$.	1 point	
By Pythagors' theorem $AC = BC = \sqrt{4^2 + 16^2} = \sqrt{272}\left(= 4\sqrt{17} \approx 16{,}49\right)$.	1 point	
The inradius *r* can be found from the relation $a = r \cdot s$ where *a* and *s* denote the area and the semiperimeter of the triangle, respectively.	1 point	
The area of the triangle *ABC* is $A_{ABC} = 64$ and	1 point	
its perimeter is: $p_{ABC} = 8\left(\sqrt{17}+1\right) \approx 40{,}98$.	1 point	
Therefore $r = \dfrac{A_{ABC}}{\dfrac{p_{ABC}}{2}} = \dfrac{2A_{ABC}}{p_{ABC}} = \dfrac{16}{\sqrt{17}+1}\left(= \sqrt{17} - 1 \approx 3{,}12\right)$.	1 point	
Total:	**8 points**	

9. a) **second solution**		
From the equation of the parabola $y = x^2 - 4x - 12 = (x-2)^2 - 16 = (x+2)(x-6)$.	1 point	
Triangle *ABC* is isosceles, its vertices can be deduced from the equation of the parabola: *A*(–2; 0), *B*(6; 0), *C*(2; –16).	1 point	
Accordingly, $AB = 8$ and the altitude to the base *AB* is $h = 16$.	1 point	
By Pythagors' theorem $AC = BC = \sqrt{4^2 + 16^2} = \sqrt{272}\left(= 4\sqrt{17} \approx 16{,}49\right)$.	1 point	
Let *O* be the incenter , denote by *E* the touching point of the incircle on the edge *AC* , finally, by *F* the midpoint of the base *AB*. The right triangles *EOC* and *FAC* are similar,	2 points	
therefore $\frac{16-r}{r} = \frac{AC}{FA} = \frac{4\sqrt{17}}{4} = \sqrt{17}$.	1 point	
Hence $r = \frac{16}{\sqrt{17}+1}\left(= \sqrt{17} - 1 \approx 3{,}12\right)$.	1 point	
Total:	**8 points**	

9. b)		
The area of the region bounded by the parabola and the x-axis (A) is equal to the opposite of the definite integral of $f:\mathbf{R}\to\mathbf{R}$, $f(x)=x^2-4x-12$ along the path between its two zeros.	2 points	*These 2 points are due even if the candidate does not write down the detailed background of the calculations but still works correctly.*
$A=-\int_{-2}^{6}\left(x^2-4x-12\right)dx=-\left[\frac{x^3}{3}-2x^2-12x\right]_{-2}^{6}=$	1 point	
$=-\left(\left(\frac{6^3}{3}-2\cdot 6^2-12\cdot 6\right)-\left(\frac{(-2)^3}{3}-2\cdot(-2)^2-12\cdot(-2)\right)\right)=$	1 point	
$=-\left((72-72-72)-\left(-\frac{8}{3}-8+24\right)\right)=$	1 point	
$=\frac{256}{3}$	1 point	
The area of the triangle ABC is $A_{ABC}=\frac{8\cdot 16}{2}=64$.	1 point	*This point is due here if the candidate has correctly calculated the area of the triangle ABC when solving part a) and the result is just repeated here.*
The proportion in question is equal to $\frac{A_{ABC}}{A}=\frac{64}{\frac{256}{3}}=\frac{3}{4}$.	1 point	
Total:	**8 points**	